Spinning of the Power Law Material

A considerable body of literature exists on the spinning and elongational viscosity of polymeric melts.[1,2] Experiments carried out under carefully controlled isothermal conditions[3,4] show elongational viscosity generally to decrease with increasing rate of elongation for many commercially important polymers. However, considerable variability from material to material is observed, and in some cases an increase in elongational viscosity with rate is indicated at low rates of elongation.

The methods used to determine the extensional behavior can be classified either as Lagrangian, which corresponds to the stretching of a tensile specimen or Eulerian, which describes the fiber spinning process. In the latter case, the measurement of thread tension, local thread velocity, and filament diameter along the spinline is required. The procedure is fraught with experimental difficulties and even if isothermal conditions are realized, morphological changes (orientation, crystallization, etc.) bring about behavior which makes generalizations difficult.

It would be instructive to consider the predictions of the rheological equation of the state of the material concerning the velocity profile and variation of elongational viscosity along the spinline. Real material behavior then can be compared to this idealized model. In this paper the behavior of a power law material in extensional flow is considered using only elementary mathematics.

When the variation in the deformation rate does not exceed 2–3 orders of magnitude, the power law model gives a good description of the shear and extensional flow behavior of polymeric melts, solutions, and superplastic metal alloys with micrograin structures. Velocity profiles along the spinline are obtained from the functional form of the equation. A single material parameter is used in the model, which corresponds to assuming isothermal conditions and no morphological changes during the attenuation process.

VELOCITY PROFILE ALONG THE SPINLINE

The rheological equation of the state of the power law material in extension is given by

$$\frac{F}{A} = \eta_1 \left(\frac{dV}{dX}\right)^n \tag{1}$$

where F is the stretching force (dyn), A is the filament cross-sectional area (cm^2), η_1 is the unit elongational viscosity, dV/dX is the axial velocity gradient (s^{-1}), and n is the power law index. The equation of continuity for incompressible material gives

$$VA = V_0 A_0 \tag{2}$$

where the subscript 0 refers to the location immediately downstream of the spinnerette after velocity rearrangement has occurred.

Substituting eq. (2) in eq. (1) and rearranging, we obtain

$$\left(\frac{F}{\eta_1 A_0 V_0}\right)^{1/n} = \frac{dV}{V^{1/n}\, dX} \tag{3}$$

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The stretching force F equals line tension for relatively slow spinning velocities. In this case the left term of eq. (3) is constant for a given power law material undergoing isothermal and isomorphic stretching.

On separating the variables and integrating with the boundary condition $V = V_0$ when X = 0, we obtain

$$\left(\frac{F}{\eta_1 A_0 V_0}\right)^{1/n} X = \frac{V^{1-1/n} - V_0^{1-1/n}}{1 - 1/n} \tag{4}$$

Equation (4) is valid for the filament all along the spinline up to the first roll station. Defining this distance where stretching stops as X_F and the velocity at that point as V_F, we can write eq. (4) as

$$\left(\frac{F}{\eta_1 A_0 V_0}\right)^{1/n} X_F = \frac{V_F^{1-1/n} - V_0^{1-1/n}}{1 - 1/n} \tag{5}$$

Dividing eq. (4) by eq. (5), we arrived at

$$\frac{X}{X_F} = \frac{(V/V_0)^{1-1/n} - 1}{(V_F/V_0)^{1-1/n} - 1} \tag{6}$$

To the extent that V_0 approximates melt velocity in spinnerette capillary, V_F/V_0 approximates jet stretch. Equation (6) gives the velocity profile for the power law material undergoing isothermal and isomorphic stretching. The results for a jet stretch of 100 × (V_F/V_0 = 100) and for various power law indices are given in Figure 1.

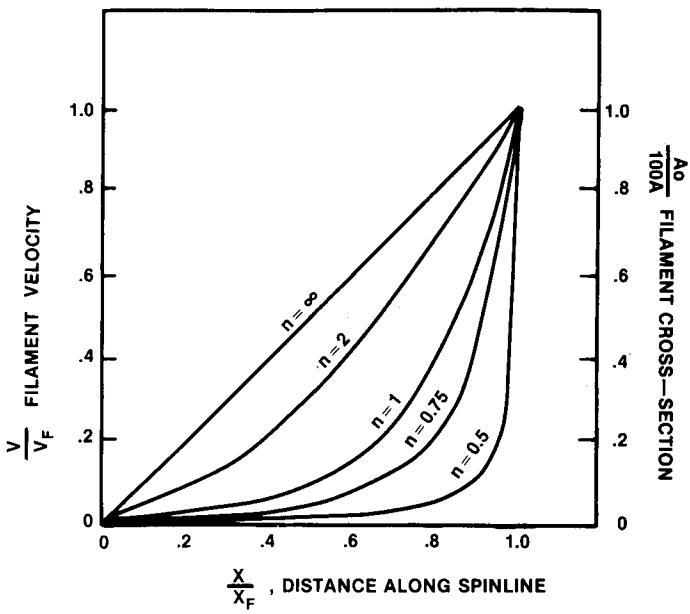

Fig. 1. Velocity profile of the power law material along the spinline.

For the Newtonian material $n=1$ and eq. (4) is indeterminate; therefore, it is necessary to start the derivation with

$$\frac{F}{A} = \eta \frac{dV}{dX} \tag{7}$$

where η is the Newtonian viscosity and is independent of the axial velocity gradient (dV/dX). The final result in this case is

$$\frac{X}{X_F} = \frac{\ln (V/V_0)}{\ln (V_F/V_0)} \tag{8}$$

The values obtained for the Newtonian material are also presented in Figure 1.

ELONGATIONAL VISCOSITY ALONG THE SPINLINE

To obtain the variation of elongational viscosity along the spinline, we begin by differentiating eq. (6), which yields

$$\frac{dX}{dV} = \frac{(1 - 1/n)X_F V^{-1/n}}{V_F^{1-1/n} - V_0^{1-1/n}} \tag{9}$$

Solving eq. (7) for η and substituting eq. (9), we arrive at

$$\eta = \frac{F}{A}\frac{dX}{dV} = \frac{F(1 - 1/n)X_F V^{-1/n}}{A(V_F^{1-1/n} - V_0^{1-1/n})} \tag{10}$$

At $X = 0$, $V = V_0$, $A = A_0$ and $\eta = \eta_0$; therefore,

$$\eta_0 = \frac{F(1-1/n)X_F V_0^{-1/n}}{A_0(V_F^{1-1/n} - V_0^{1-1/n})} \tag{11}$$

Dividing eq. (10) by eq. (11) and noting that $AV = A_0V_0$, we obtain

$$\frac{\eta}{\eta_0} = \frac{A_0}{A}\left(\frac{V}{V_0}\right)^{-1/n} = \left(\frac{V}{V_0}\right)^{1-1/n} \tag{12}$$

Substituting the value of $\left(\frac{V}{V_0}\right)^{1-1/n}$ from eq. (6) and after simplifying, we arrive at eq. (13), which gives the variation of elongational viscosity along the spinline:

$$\frac{\eta}{\eta_0} = \frac{X}{X_F}\left[\left(\frac{V_F}{V_0}\right)^{1-1/n} - 1\right] + 1 \tag{13}$$

The results obtained from eq. (13) for a jet stretch of $100 \times (V_F/V_0 = 100)$ and for various values of power law indices are given in Figure 2. The equation does not become indeterminate for $n=1$ and shows that for the Newtonian material the viscosity is constant along the spinline. A derivation beginning with eq. (8) also results in the strain rate independence and the spinline

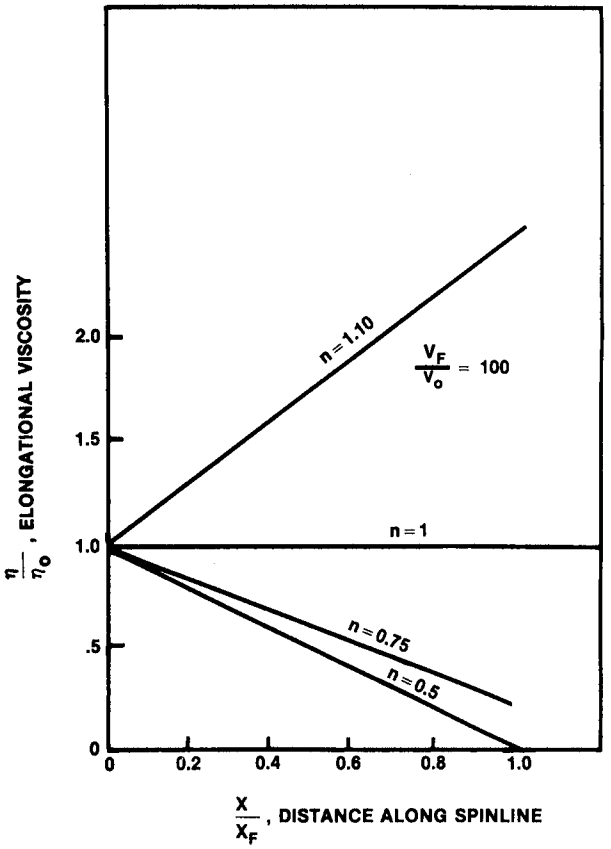

Fig. 2. Elongational viscosity along the spinline for the power law material.

constancy of the Newtonian material. Equation (13) also shows that the viscosity should decrease along the spinline when the power law index (n) is less than 1 (strain thinning), while for the strain thickening materials ($n > 1$) the elongational viscosity increases along the spinline.

DISCUSSION AND IMPLICATIONS FOR FIBER SPINNING

A polymer solution or melt is termed "spinnable" if long threads can be formed when a rod is brought in contact with it and then withdrawn. While some authors have associated this property with elasticity, others have suggested that an increase in elongational viscosity along the spinline is required for a material to be spinnable. According to the latter view, it would appear that strain thinning materials should not be spinnable under isothermal spinning conditions. This is contrary to experience since most of the polymers used in fiber spinning exhibit shear and strain thinning behavior. Undoubtedly, filament cooling during melt spinning, coagulation, or solvent evaporation in solution spinning are factors that greatly increase viscosity along the spinline. However, such an increase is not a necessary condition to achieve spinnability as shown in isothermal spinning experiments using strain thinning materials.[4]

Ultimately what determines the length of a liquid filament in a "spinnability" test is filament breakage. The viscosity of materials at spinning temperature is sufficiently large so that surface tension can be ruled out as a failure mechanism. In this case what causes filament breakage is not low viscosity but the attainment of a critical level of stress in the narrowest cross section of the filament. The velocity profiles given in Figure 1 show that the attenuation of the filament occurs further away from the spinnerette as the power law index decreases. On this basis strain thinning materials should form longer threads and therefore have greater "spinnability." Although in the spinnability test the weight of the withdrawn filament is

greater for the strain thinning material, this factor is not large enough to change the final result.

In commercial practice filament spinning is generally downwards except when a coagulation bath is used. In this case the stress on the filament, ignoring drag forces, has one component due to filament weight and another due to the applied tension. While the first has its maximum at the spinnerette, the second has its maximum at the takeup roll. The combination reduces the axial stress gradient along the spinline and provides greater stretchability than would be possible otherwise.

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